

Cranking mass parameters for fission

M. Mirea¹ and R.C. Bobulescu²

¹Horia Hulubei National Institute for Physics and Nuclear Engineering, Department of Theoretical Physics, P.O. Box MG 6, Bucharest, Romania

²University of Bucharest, Faculty of Physics, P.O. Box MG-11, Bucharest, Romania

Abstract. A formalism for semi-adiabatic cranking mass parameters is presented. For the fission process of ^{234}U , the time-dependent pairing equations of motion were used to calculate the excitation energy and to extract values of the cranking inertia. A fission barrier is determined by minimizing the action trajectory in a five dimensional configuration space spanned by elongation, necking, deformations of fragments and mass-asymmetry. The deformation energy is computed in the the frame of the microscopic-macroscopic model. The two center shell model with Woods-Saxon potentials is used in this context. Values of the inertia for excited fissioning systems are reported. A dependence between the cranking mass parameters and the intrinsic excitation energy is evidenced.

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1. Introduction

In microscopic-macroscopic treatments of nuclear fission [1], the whole nuclear system is characterized by some collective coordinates associated with some degrees of freedom that determine approximately the behavior of many intrinsic variables. The basic ingredients in such an analysis is a shape parametrization that depends on several macroscopic degrees of freedom. The generalized coordinates of deformation vary in time leading to a split of the nuclear system in two separated fragments. Thus, these coordinates describe the change in time of the average field. Traditionally, in the large amplitude collective motion like fission or fusion, the calculation of the mass parameters are made within the cranking model [2, 3]. These parameters are calculated adiabatically. In reality, the nucleus can be excited with respect the ground state at any deformation. The excitation energy is shared between the collective kinetic energy of the fragments and the intrinsic energy released finally through gamma-ray and neutron emission. As mentioned in Ref. [4], no unique separation between these two kind of dumping can be realized. Several attempts were realized in order to investigate the change of the cranking mass parameters as function of the intrinsic excitation energy. In this respect, investigation based on the cranking model have been done for the rotational motion [5] and for the behavior of the temperature dependent mass parameter [6, 7, 8]. A theory for non-adiabatic cranking was proposed in Ref. [9] where the time-dependent many-body Schrodinger equation is solved and the time dependence of the collective motion is determined with the classical Lagrange equation of motion. It is worth to mention that in Ref. [10] an expression is derived for the collective kinetic energy containing corrections up to the fourth-order in the collective velocities. The fourth-corrections to the cranking approximation at hyperdeformations were found very large. The authors concluded that the perturbation treatment when applied to fission is invalidated.

In this paper, a new treatment for the cranking model is realized that takes into account the intrinsic excitation energy. In the next section, a semi-adiabatic cranking approximation is derived for even-even systems to investigate the general features of the cranking mass parameters when the nucleus is internally excited. To make the problem tractable, it is assumed that the system deforms slowly in time. Therefore, the matrix elements of the time-derivative of the wave functions are smalls. Because these matrix elements are responsible for quasi-particle excitations, our approximation allows to consider the system in a seniority-zero state during fission. In other words, the contributions to the inertia originating from seniority-two configurations are considered negligible. In section 3, the mass parameters are calculated. To obtain the inertia, a least action trajectory for the evolution of the nuclear system is determined. The single-particle energies for protons and neutrons were calculated along this path. Then, the intrinsic excitation energy of the fissioning system is evaluated within the time-dependent pairing equations of motion using different values of the inter-nuclear velocities. At each deformation, instantaneous values of the single-particle densities and

of the pairing moment components are deduced. The semi-adiabatic effective mass is evaluated using these values. Some behavior concerning the dependence of the cranking mass parameters versus the intrinsic excitation energy are extracted. In the last section, a discussion is made.

2. Formalism

The single particle motion of a particle in an average field is governed by the Schrödinger equation that includes a Hamiltonian with pairing residual interactions. This Hamiltonian depends on some time-dependent collective parameters $q(t) = \{q_\nu(t)\}$ ($\nu = 1, \dots, n$), such as the internuclear distances between the nascent fragments, the mass-asymmetry, the fragment deformations or the necking parameter:

$$H(t) = \sum_{k>0} \epsilon_k [q(t)] (a_k^+ a_k + a_{\bar{k}}^+ a_{\bar{k}}) - G \sum_{k,i>0} a_k^+ a_{\bar{k}}^+ a_i a_{\bar{i}}. \quad (1)$$

where ϵ_k are single-particle energies, G is an monopole pairing interaction constant and a_k^+ denote creation operators. In order to obtain the time dependent pairing equations of motion, we shall start from the variational principle taking the following energy functional

$$\mathcal{L} = \langle \varphi | H - i\hbar \frac{\partial}{\partial t} - \lambda \hat{N} | \varphi \rangle \quad (2)$$

and by assuming the many-body state formally expanded as a superposition of time dependent BCS seniority-zero and seniority-two adiabatic wave functions

$$| \varphi(t) \rangle = c_0(t) | \phi_{\text{BCS}} \rangle + \sum_{j,l} c_{jl}(t) \alpha_j^+ \alpha_l^+ | \phi_{\text{BCS}} \rangle \quad (3)$$

where

$$| \phi_{\text{BCS}} \rangle = \prod_k (u_k + v_k a_k^+ a_{\bar{k}}^+) | 0 \rangle \quad (4)$$

is the seniority-zero Bogoliubov wave function. The excited seniority-two configurations are obtained by mean of quasiparticle creation and annihilation operators:

$$\begin{aligned} \alpha_k &= u_k a_k - v_k a_{\bar{k}}^+; & \alpha_{\bar{k}} &= u_k a_{\bar{k}} + v_k a_k^+; \\ \alpha_k^+ &= u_k a_k^+ - v_k^* a_{\bar{k}}; & \alpha_{\bar{k}}^+ &= u_k a_{\bar{k}}^+ + v_k^* a_k; \end{aligned} \quad (5)$$

In definitions (2) and (3), c_0 and c_{jl} are amplitudes of the two kinds of configurations, λ is the chemical potential, and \hat{N} is the particle number operator. Because only the relative phase between the parameters u_k (vacancy amplitudes) and v_k (occupation amplitudes) matters, in the following u_k is considered to be a real quantity and v_k a complex one. To minimize the functional, the expression (2) is derived with respect the independent variables v_k , v_k^* , c_0 , c_{jl} (the amplitudes of the wave function), together with their complex conjugates, and the resulting equations are set to zero.

As deduced in Refs. [11, 12] and as detailed in the Appendix, for a seniority-zero nuclear system, the pairing equations of motion are:

$$\begin{aligned} i\hbar \dot{\rho}_k &= \kappa_k \Delta_0^* - \kappa_k^* \Delta_0; \\ i\hbar \dot{\kappa}_k &= (2\rho_k - 1) \Delta_0 + 2\kappa_k (\epsilon_k - \lambda) - 2G \rho_k \kappa_k; \end{aligned} \quad (6)$$

This system is sometimes called time dependent Hartree-Fock-Bogoliubov equations [12]. These equations were already used to determine the intrinsic excitation energy in fission [13, 14] or to investigate the pair-breaking mechanism [15]. The following notations are used in Eqs. (6) and in the remaining part of the article:

$$\begin{aligned}\Delta_0 &= G \sum_k \kappa_k; \Delta_{jl} = G \sum_{k \neq j, l} \kappa_k; \\ \kappa_k &= u_k v_k; \quad \rho_k = |v_k|^2;\end{aligned}\tag{7}$$

where Δ_0 is the gap parameter for the seniority-zero state while Δ_{jl} are related to seniority-two states. κ_k are pairing moment components and ρ_k are single particle densities.

An estimate of the intrinsic seniority-zero state excitation energy [11] can be obtained with the relation

$$E^* = E_0 - E_{\text{BCS}}\tag{8}$$

where

$$E_0 = \langle \phi_{\text{BCS}} | H - \lambda \hat{N} | \phi_{\text{BCS}} \rangle = 2 \sum_k \rho_k (\epsilon_k - \lambda) - \frac{|\Delta_0|^2}{G} - G \sum_k \rho_k^2;\tag{9}$$

is the expected value of the Hamiltonian (1) for the seniority-zero state and E_{BCS} is the stationary energy obtained by replacing κ_k and ρ_k within the time-independent BCS parameters $\tilde{\kappa}_k$ and $\tilde{\rho}_k$ in formula (9).

As indicated in the Appendix, the next semi-adiabatic cranking formula can be obtained for the effective mass parameters B :

$$\begin{aligned}B_{\nu\mu} &= B_{1\nu\mu} + B_{2\nu\mu} \\ &= 2\hbar^2 \sum_{m, n \neq m} \frac{(E_{mn} - E_0) \left| \frac{\kappa_m \sqrt{\rho_m} \kappa_n}{|\kappa_m| \sqrt{\rho_n}} - \frac{\kappa_n \sqrt{\rho_n} \kappa_m}{|\kappa_n| \sqrt{\rho_m}} \right|^2 \langle m | \frac{\partial H}{\partial q_\nu} | n \rangle \langle n | \frac{\partial H}{\partial q_\mu} | m \rangle}{(E_{mn} - \sum_{k \neq m, n} T_k - E_0 + \sum_k T_k)^2 (\epsilon_m - \epsilon_n)^2} \\ &+ 2\hbar^2 \sum_m \frac{(E_{mm} - E_0) \left(\frac{\kappa_m}{\rho_m} \frac{\partial \rho_m}{\partial q_\nu} - \frac{\kappa_m^*}{\kappa_m^*} \frac{\partial \kappa_m^*}{\partial q_\nu} \right) \left(\frac{\kappa_m^*}{\rho_m} \frac{\partial \rho_m}{\partial q_\mu} - \frac{\kappa_m^*}{\kappa_m^*} \frac{\partial \kappa_m^*}{\partial q_\mu} \right)}{(E_{mm} - \sum_{k \neq m} T_k + T_m - E_0 + \sum_k T_k)^2}\end{aligned}\tag{10}$$

where the values of ρ_k and κ_k are solutions of the time dependent pairing equations (6)i and the index ν is associated to the collective coordinate q_ν . E_{jl} are exactly the expected values of the Hamiltonian (1) for seniority-two configurations:

$$E_{jl} = \langle \alpha_j^+ \alpha_l^+ \phi_{\text{BCS}} | H - \lambda \hat{N} | \alpha_j^+ \alpha_l^+ \phi_{\text{BCS}} \rangle\tag{11}$$

$$= 2 \sum_{k \neq j, l} \rho_k (\epsilon_k - \lambda) - \frac{|\Delta_{jl}|^2}{G} - G \sum_{k \neq j, l} \rho_k^2 + |\epsilon_j - \lambda| + |\epsilon_l - \lambda|;$$

$$E_{jj} = \langle \alpha_j^+ \alpha_j^+ \phi_{\text{BCS}} | H - \lambda \hat{N} | \alpha_j^+ \alpha_j^+ \phi_{\text{BCS}} \rangle = 2 \sum_{k \neq j} \rho_k (\epsilon_k - \lambda)\tag{12}$$

$$+ 2u^2 |\epsilon_j - \lambda| - \frac{|\Delta_j|^2}{G} + \kappa_j \Delta_{jj} + \kappa_j^* \Delta_{jj}^* - G \sum_{k \neq j} \rho_k^2 + |\kappa_j|^4 / \rho_j^2;$$

and T_k are energy terms associated to single-particle states:

$$T_k = 2\rho_k(\epsilon_k - \lambda) - 2G\rho_k^2 + \frac{\kappa_k\Delta_0^* + \kappa_k^*\Delta_0}{2} \left(\frac{\rho_k^2}{|\kappa_k|^2} - 1 \right); \quad (13)$$

The mass parameters can be obtained from formula (10) only if the derivatives $\partial\rho_m/\partial q_\nu$ and $\partial\kappa_m/\partial q_\nu$ are supplied. In the BCS stationary state, the derivatives $\partial\tilde{\rho}_m/\partial q_\nu$ and $\partial\tilde{\kappa}_m/\partial q_\nu$ depend only on the collective variables q_ν and the cranking formula can be easily obtained. On the other hand, if the system follows a trajectory in the configuration space and the motion is characterized by some given collective velocities, then the derivatives depend also on \dot{q}_ν . Therefore, the second term in Eq. (10) depends on the history of the system and can be calculated only if the variations of ρ_k and κ_k are known. The inertia along the trajectory is

$$\begin{aligned} B &= \sum_{\nu,\mu} B_{\nu\mu} \frac{\partial q_\nu}{\partial R} \frac{\partial q_\mu}{\partial R} \\ &= \sum_{\nu,\mu} B_{1\nu\mu} \frac{\partial q_\nu}{\partial R} \frac{\partial q_\mu}{\partial R} + 2\hbar^2 \sum_m \frac{(E_{mm} - E_0) \left| \frac{\kappa_m}{\rho_m} \frac{\partial \rho_m}{\partial R} - \frac{\kappa_m}{\kappa_m^*} \frac{\partial \kappa_m^*}{\partial R} \right|^2}{(E_{mm} - \sum_{k \neq m} T_k + T_m - E_0 + \sum_k T_k)^2} \end{aligned} \quad (14)$$

where one coordinate R is taken as the independent variable. The other coordinates q_ν are taken to be function of R [5]. From definition, the collective kinetic energy is $E_c = B\dot{R}^2/2$ and the last term in Eq. (14) becomes:

$$E_{c0} = \hbar^2 \sum_m \frac{(E_{mm} - E_0) \left| \frac{\kappa_m}{\rho_m} \dot{\rho}_m - \frac{\kappa_m}{\kappa_m^*} \dot{\kappa}_m^* \right|^2}{(E_{mm} - \sum_{k \neq m} T_k + T_m - E_0 + \sum_k T_k)^2} \quad (15)$$

This term depends only on the derivatives with respect to time $\dot{\kappa}_m$ and $\dot{\rho}_m$. Their expressions are given by the coupled channel system of equations (6). That means, the term (15) is practically independent on the collective velocity and depends only on the values of ϵ_m , κ_m and ρ_m , that is on the structure of the system and its intrinsic excitation and represents the minimal collective kinetic energy. Therefore, the term (15) can be viewed as an ground collective kinetic energy. In other words, if the nucleus is internally excited and the derivatives of the probabilities $\dot{\kappa}$, $\dot{\rho}$ are different from zero, the system possesses a minimal collective kinetic energy. The term (15) can be considered as responsible for energy transfer between intrinsic and collective degrees of freedom.

3. Results

The calculation addresses the fission of ^{234}U . As already mentioned, the basic ingredient in our analysis is the nuclear shape parametrization. The nuclear shape parametrization used in the following is given by two ellipsoids of different sizes smoothly joined by a third surface obtained [16] by the rotation of a circle around the axis of symmetry. Five degrees of freedom characterize this parametrization: the elongation given by the inter-nuclear distance R between the centers of the ellipsoids, the two deformations of the nascent fragments characterized by their eccentricities $\varepsilon_i = \sqrt{1 - b_i^2/a_i^2}$ ($i = 1, 2$), the mass asymmetry given by the ratio between the major semi-axis of the fragments

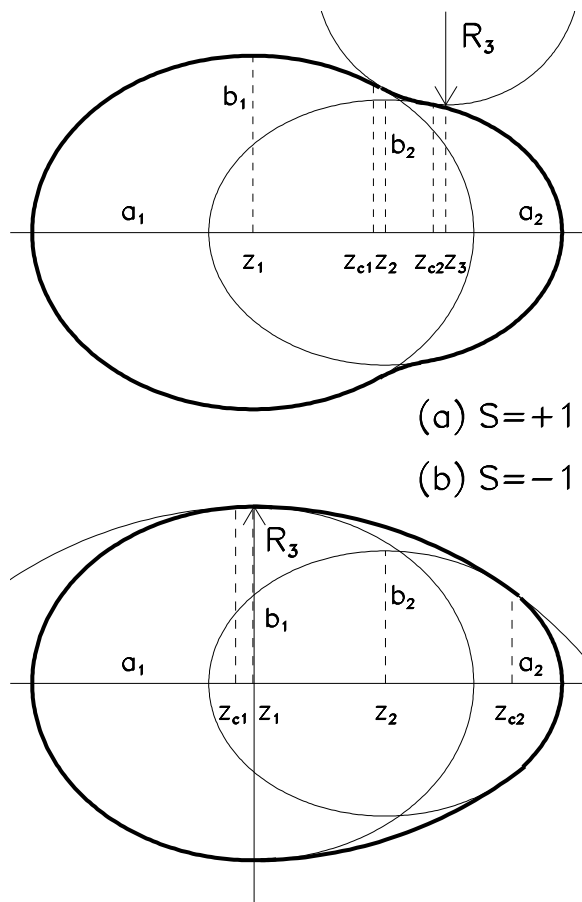


Figure 1. Nuclear shape parametrization.

$\eta = a_1/a_2$, and the necking parameter related to the median curvature $C = s/R_3$ (R_3 being the radius of the intermediate circle and S the sign associated to the curvature). The meaning of all geometric parameters can be understood by inspecting the Fig. 1.

As specified in Ref. [5], first of all, a calculation of the fission trajectory in our five-dimensional configuration space, beginning with the ground-state of the system up to the exit point of the barrier must be performed. This can be done by minimizing the action integral. For this purpose, two ingredients are required: the deformation energy V and the tensor of the effective mass. The deformation energy was obtained [1] by summing the liquid drop energy with the shell and the pairing corrections. The macroscopic energy is obtained in the framework of the Yukawa plus exponential model [17] extended for binary systems with different charge densities [18]. The Strutinsky microscopic corrections were computed on the basis of the Woods-Saxon superasymmetric two-center shell model [16]. This model gives the single particle level diagrams by diagonalizing a Woods-Saxon potential, corrected within spin-orbit and Coulomb terms, in the analytic eigenvalue basis of the two center semi-symmetric harmonic model [19, 20]. The effective mass is computed within the cranking adiabatic approximation as given in [5]. After minimization, the dependences between the generalized coordinates q_ν ($\nu = 1, \dots, 5$) in the

region comprised between the parent ground state configuration and the exit point of the external fission barrier supply the least action trajectory. The ground-state corresponds to the lowest deformation energy in the first well. The least action trajectory is obtained within a numerical method. Details about the numerical procedure of minimization and about the model can be found in Refs. [14, 16, 21] and references therein. Plots of the minimal deformation energy surface as function of the necking coordinate C and the elongation R are displayed in Figs. 2 (a) and (b). In Fig. 2 (b) the minimal action trajectory is also plotted. The resulting ^{234}U fission barrier is plotted on Fig. 2 (c) as function of the distance between the centers of the nascent fragments R . Some nuclear shapes obtained along the minimal action trajectory are inserted in the plot.

The realistic level scheme along the least action trajectory were also obtained within the superasymmetric Woods-Saxon two-center shell model. Within the energy diagrams, the system (6) is solved having as initial conditions the stationary BCS state in the ground state configuration. Three values of the internuclear velocity \dot{R} were taken into account: 1×10^4 , 1×10^5 and 1×10^6 fm/fs, corresponding to a time to penetrate the barrier comprised between $[10^{-18}, 10^{-20}]$ s. In Fig. 3 (a), the intrinsic excitation energy given by Rel. (8) is plotted as function of the internuclear distance for the three values of \dot{R} . If the internuclear velocity increases, the excitation energy becomes larger.

The inertia along the trajectory was calculated within three different approaches: the adiabatic cranking model of Ref. [5, 22, 23], the formula (14) within stationary BCS parameters $\tilde{\kappa}$ and $\tilde{\rho}$ and the same formula within κ and ρ values given by Eq. (6). The total effective mass is the sum of partial values obtained for neutron and proton subsystems.

In Fig. 3 (c), the inertia along the minimal action path calculated within the adiabatic cranking model is displayed with a thick line. In the ground state of the first well, the inertia is very small. Large values of the cranking inertia are obtained around the exit point of the outer fission barrier, close to the scission configuration. A value approaching the reduced mass is obtained after the scission point. A similar behavior, exhibiting very large values of the inertia around the touching configuration, was obtained for light systems in Ref. [24], where a version of the Woods-Saxon two center shell model based on the molecular orbital approach is used. In the same plot, a thin line gives the inertia obtained within formula (14) by using the stationary values $\tilde{\kappa}$ and $\tilde{\rho}$. This semi-adiabatic inertia exhibits a similar shell structure as the adiabatic cranking model. The semi-adiabatic model gives in general lower values than the adiabatic cranking one, excepting the regions of the ground state and of the scission point where the values are larger.

In Fig. 3 (b) a comparison is made between inertia computed within relation (14) for different values of the internuclear velocity. The same values of the internuclear velocities \dot{R} were taken as those used to determine the excitation energies. The inertia along the minimal action trajectory has the largest values for the lowest \dot{R} taken into consideration. In general, the values obtained for $\dot{R} = 1 \times 10^4$ fm/fs represent an upper limit for the magnitude of the inertia. It must be mentioned that for this collective

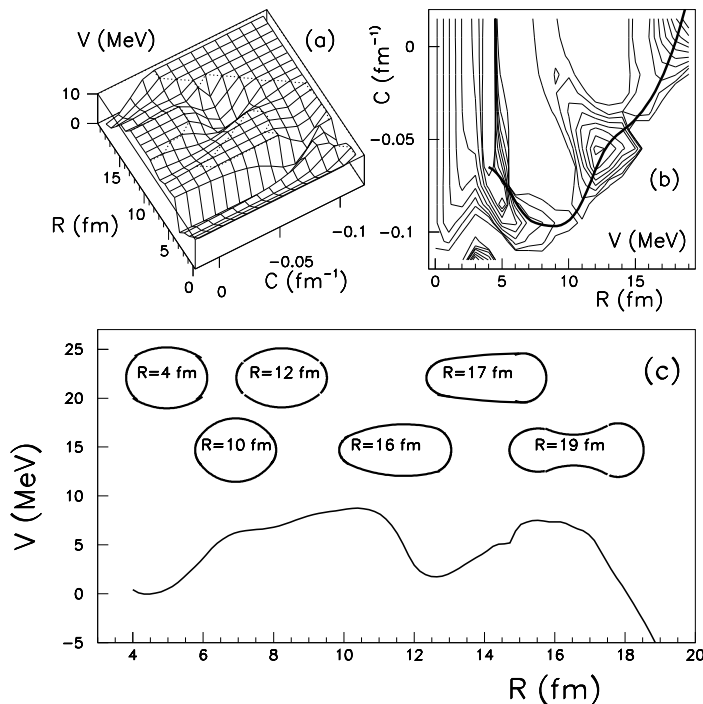


Figure 2. (a) Minimal values of the deformation energy in MeV as function of the necking coordinate C and the elongation R for ^{234}U . (a) Contours of the deformation energy in step of 1 MeV. The least action trajectory is superimposed. (c) Potential barrier. Some shapes obtained during the fission process together with the values of the elongation R are inserted.

velocity, the calculated intrinsic excitation energy is practically negligible. For higher values of \dot{R} , when the excitation energy becomes important, the magnitude of the inertia is considerably lower. The values obtained for $\dot{R} = 1 \times 10^5$ fm/fs and $\dot{R} = 1 \times 10^6$ fm/fs are situated in the vicinity of those obtained within the stationary values. For the three collective velocities, the shell structure resemble.

4. Discussion

In Ref. [8], a temperature dependent cranking model that includes a parameter for the dissipation was used. It was shown that the increase of the temperature smoothen and lower the mass parameter. In the high temperature limit, the cranking results are quite close to the irrotational flow results. Our calculations also show that the inertia decreases when the excitation energy increases.

Several approximations were derived in the literature to compute the collective inertia. In the frame of the response theory [25] the Hamiltonian is expanded around a particular value of the macroscopic coordinate and the first derivative is treated with the time dependent perturbation approach. Another method is related to the generator coordinate method proposed in Ref. [26] within the Gaussian Overlap Approximation (GOA). This method allows to obtain a representation of any operator in the collective

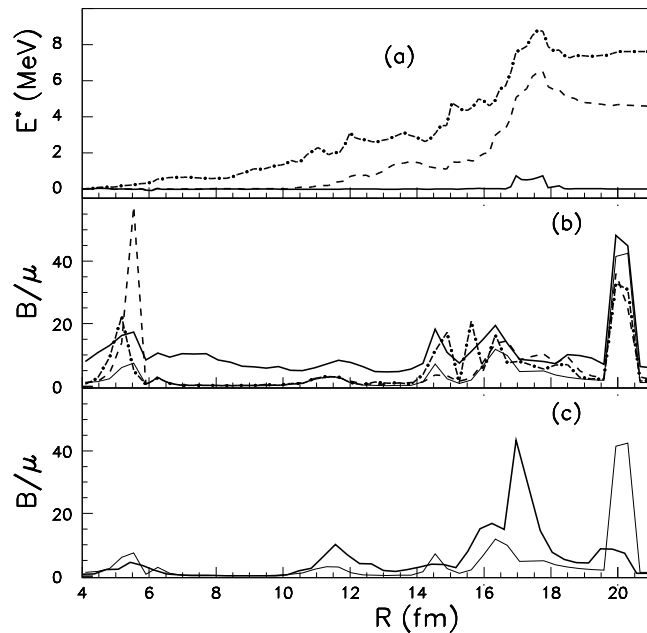


Figure 3. (a) Intrinsic excitation energies along the minimal action trajectory as function of the elongation R for three values of the internuclear distance velocity \dot{R} : full line $\dot{R} = 1 \times 10^4$ fm/fs, dashed line $\dot{R} = 1 \times 10^5$ fm/fs and dot-dashed line $\dot{R} = 1 \times 10^6$ fm/fs. (b) The thick lines represent the inertia B divided by the reduced mass μ for the three velocities taken into consideration as function of R . The same line types are used as in panel (a) for the values of the velocities. The thin line corresponds to the inertia computed within BCS parameters $\tilde{\kappa}$ and $\tilde{\rho}$. (c) Comparison between the inertia calculated within formula (14) with stationary values $\tilde{\kappa}$ and $\tilde{\rho}$ (thin line) and the classical cranking formula [5] (thick line). Asymptotically, the two inertia reaches approximately the reduced mass μ .

space. In general, the GOA mass is about 2/3 times smaller than the adiabatic cranking mass [27] but both quantities exhibits a similar shell structure. Adiabatic mass parameters for fission were derived also from the Time-Dependent Hartree-Fock-Bogololiubov (TDHFB) theory [28]. A comparison between values of the collective mass tensor obtained with three different models, i.e., cranking, GOA and adiabatic TDHFB, showed that the adiabatic TDHFB mass exhibits more pronounced variations than the cranking and the GOA masses. As expected, the GOA gives smaller values than the cranking model. Our results show that for stationary values of $\tilde{\kappa}$ and $\tilde{\rho}$, the inertia is lower than the adiabatic cranking values.

In Ref. [29] the second 0^+ collective energy level was calculated for the rare-earth and actinide nuclei using the Bohr-Sommerfeld quantization rule. The results overestimates the experimental values in average within a factor 2. This discrepancy can be caused either by the shape of the potential in the ground state or either by a systematic too low value of the cranking inertia. In our calculations, in the first well has larger values than that obtained within the adiabatic cranking model.

In conclusion, a semi-adiabatic formalism based on the time-dependent pairing equations was described. The values of the inertia obtained within this model exhibit a strong dependence on the intrinsic excitation energy. In general, the semi-adiabatic inertia have a similar shell structure as the adiabatic cranking model.

5. Acknowledgments

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6. APPENDIX

In order to simplify the calculations, the variations of the parameter u_k and v_k as function of the seniority number, called blocking effect, is neglected. A treatment involving the blocking effect was realized in Ref. [16] in order to generalize the Landau-Zener effect and in Ref. [15] to evidence a new dynamical pair breaking effect. Within the wave function (3), using the notation (4) and the following identities

$$\langle c_0 \phi_{\text{BCS}} | \frac{\partial}{\partial t} | c_0 \phi_{\text{BCS}} \rangle = c_0^* \dot{c}_0 + |c_0|^2 \sum_k (u_k \dot{u}_k + v_k^* \dot{v}_k); \quad (16)$$

$$\begin{aligned} \langle \sum_{j,i \neq j} c_{ji} \alpha_j^+ \alpha_i^+ \phi_{\text{BCS}} | \frac{\partial}{\partial t} | \sum_{k,l \neq k} c_{kl} \alpha_k^+ \alpha_l^+ \phi_{\text{BCS}} \rangle &= \sum_{j,l \neq j} [c_{jl}^* c_{jl} + |c_{jl}|^2 \sum_{k \neq j,l} (u_k \dot{u}_k + v_k^* \dot{v}_k)] \\ &+ \sum_{j,i \neq j} \sum_{k,l \neq k} c_{ji}^* c_{kl} [(u_k u_j + v_k^* v_j) \langle j | \frac{\partial}{\partial t} | k \rangle \delta_{il} \\ &+ (u_i u_l + v_i^* v_l) \langle \bar{i} | \frac{\partial}{\partial t} | \bar{l} \rangle \delta_{jk}]; \end{aligned} \quad (17)$$

$$\langle c_{jl} \alpha_j^+ \alpha_l^+ \phi_{\text{BCS}} | \frac{\partial}{\partial t} | c_0 \phi_{\text{BCS}} \rangle = c_{jl}^* c_0 (u_j v_l - v_j u_l) \langle j | \frac{\partial}{\partial t} | l \rangle; \quad (18)$$

$$\langle c_0 \phi_{\text{BCS}} | \frac{\partial}{\partial t} | c_{jl} \alpha_j^+ \alpha_l^+ \phi_{\text{BCS}} \rangle = -c_0^* c_{jl} (u_j v_l^* - v_j^* u_l) \langle j | \frac{\partial}{\partial t} | l \rangle; \quad (19)$$

$$\langle c_{jl} \alpha_j^+ \alpha_j^+ \phi_{\text{BCS}} | \frac{\partial}{\partial t} | c_0 \phi_{\text{BCS}} \rangle = -c_{jl}^* c_0 (u_j \dot{v}_j - v_j \dot{u}_j); \quad (20)$$

$$\langle c_0 \phi_{\text{BCS}} | \frac{\partial}{\partial t} | c_{jj} \alpha_j^+ \alpha_j^+ \phi_{\text{BCS}} \rangle = -c_0^* c_{jj} (u_j \dot{v}_j^* - v_j^* \dot{u}_j); \quad (21)$$

the energy functional (2) becomes

$$\begin{aligned} \mathcal{L} = & |c_0|^2 \left\{ \sum_k 2 |v_k|^2 (\epsilon_k - \lambda) - G \left| \sum_k u_k v_k \right|^2 - G \sum_k |v_k|^4 \right\} \\ & + \sum_{j,l \neq j} |c_{jl}|^2 \left\{ \sum_{k \neq j,l} 2 |v_k|^2 (\epsilon_k - \lambda) + |\epsilon_j - \lambda| + |\epsilon_k - \lambda| \right\} \end{aligned}$$

$$\begin{aligned}
& -G \left| \sum_{k \neq j, l} u_k v_k \right|^2 - G \sum_{k \neq j, l} |v_k|^4 \Big\} \\
& + \sum_j |c_{jj}|^2 \left\{ \sum_{k \neq j} 2 |v_k|^2 (\epsilon_k - \lambda) + 2u^2 |\epsilon - \lambda_{jj}| \right. \\
& - G \left| \sum_{k \neq j} u_k v_k \right|^2 + u_j v_j G \sum_{k \neq j} (u_k v_k) + u_j v_j^* G \sum_{k \neq j} (u_k v_k^*) - G \sum_{k \neq j} |v_k|^4 - u_j^4 \Big\} \\
& - i\hbar \left\{ c_0^* \dot{c}_0 + |c_0|^2 \sum_k \frac{1}{2} (v_k^* \dot{v}_k - \dot{v}_k^* v_k) \right. \\
& + \sum_{j, l \neq j} \left[c_{jl}^* \dot{c}_{jl} + |c_{jl}|^2 \sum_{k \neq j, l} \frac{1}{2} (v_k^* \dot{v}_k - \dot{v}_k^* v_k) \right] \\
& + \sum_j \left[c_{jj}^* \dot{c}_{jj} + |c_{jj}|^2 \left[\sum_{k \neq j} \frac{1}{2} (v_k^* \dot{v}_k - \dot{v}_k^* v_k) - \frac{1}{2} (v_j^* \dot{v}_j - \dot{v}_j^* v_j) \right] \right] \\
& + \sum_{j, l \neq j} \left[c_{jl}^* c_0 (u_j v_l - v_j u_l) + c_{jl} c_0^* (v_j^* u_l - v_l^* u_j) \right] \langle j | \frac{\partial}{\partial t} | l \rangle \\
& + \sum_j \left[c_{jj}^* c_0 (-u_j \dot{v}_j + v_j \dot{u}_j) + c_{jj} c_0^* (-u_j \dot{v}_j^* + v_j^* \dot{u}_j) \right] \\
& + \sum_{j, l \neq j} \sum_{m, n \neq m} c_{jl}^* c_{mn} \left[(u_m u_j + v_m^* v_j) \langle j | \frac{\partial}{\partial t} | m \rangle \delta_{ln} \right. \\
& \left. + (u_n u_l + v_n^* v_l) \langle \bar{l} | \frac{\partial}{\partial t} | \bar{n} \rangle \delta_{jm} \right] \Big\}, \tag{22}
\end{aligned}$$

where the dot represents the time derivative. In order to obtain the equations of motion, the previous expression must be derived with respect the independent variables and their complex conjugates and the resulting expressions must be set to zero. The assumption of a slowly varying deformation with respect the time will be used. In these circumstances, the probability to find the system in a real or virtual seniority-two state is much lower than the probability to find the system in a seniority-zero one. These statements can be translated in the following conditions: $|c_{ij}|^2 \ll 1$ and $|c_0|^2 \approx 1$. In this context, as prescribed in Ref. [16] and using the derivatives with respect v_l and v_l^* , two equations are obtained:

$$-i\hbar \dot{v}^* = 2v_m^* (\epsilon_m - \lambda) - G \left[\sum \kappa_k \left(-\frac{v_l^* v_l^*}{2u_l} \right) + \left(u_l - \frac{\rho_l}{2u_l} \right) \sum \kappa_k^* + 2\rho_l v_l^* \right], \tag{23}$$

and another for its complex conjugate. The notations (7) are used. From Eq. (23) and its complex conjugate, the system (6) follows and the expressions involving the product between v_k and \dot{v}_k^* appearing in equation (22) are determined:

$$T_k = \frac{i\hbar}{2} (\dot{v}_l v_l^* - \dot{v}_l^* v_l) = 2\rho_l (\epsilon_l - \lambda) - G\rho_l^2 + \frac{\Delta_0^*}{2} \left(\frac{\rho_l^2}{\kappa_l^*} - \kappa_l \right) + \frac{\Delta_0}{2} \left(\frac{\rho_l^2}{\kappa_l} - \kappa_l^* \right). \tag{24}$$

The discussion found Refs. [11] reveal that the approximation used to obtain Rel. (23) helps us to describe approximately the effect of the residual interaction on dissipation

and we eliminate only the terms related to the collective kinetic energy. The collective kinetic energy will be treated separately by solving equations involving different seniority states.

In order to determine the excitations between configurations the equations (22) must be derived with respect the amplitudes c_0 , c_0^* , c_{ij} , c_{ij}^* , c_{jj} and c_{jj}^* . The next three equations follows:

$$c_0 E_0 + i\hbar \dot{c}_0 - c_0 \sum_k T_k - i\hbar \sum_{l,j \neq l} c_{jl} (u_j v_l^* - v_j^* u_l) \langle j | \frac{\partial}{\partial t} | l \rangle = 0; \quad (25)$$

$$c_{jl} E_{jl} - i\hbar \dot{c}_{jl} - c_{jl} \sum_{k \neq j,l} T_k - i\hbar [c_0 (-u_j v_l + v_j u_l) \langle j | \frac{\partial}{\partial t} | l \rangle \quad (26)$$

$$+ \sum_m c_{ml} (u_j u_m + v_j v_m^*) \langle m | \frac{\partial}{\partial t} | j \rangle + \sum_n c_{jn} (u_n u_l + v_l v_n^*) \langle \bar{n} | \frac{\partial}{\partial t} | \bar{l} \rangle = 0;$$

$$c_{jj} (E_{jj} - \sum_{k \neq j} T_k + T_j) - i\hbar \dot{c}_{jj} + i\hbar c_0 (u_j \dot{v}_j - v_j \dot{u}_j) = 0; \quad (27)$$

and three equations for their complex conjugates. The notations (9), (11), (12), (13) and (24) were used. The homogeneous solutions are:

$$c_{0(h)}(t) = c_{0(h)}(0) \exp \left(-\frac{i}{\hbar} \int_0^t (E_0 - \sum_k T_k) dt \right); \quad (28)$$

$$c_{jl(h)}(t) = c_{jl(h)}(0) \exp \left(-\frac{i}{\hbar} \int_0^t (E_{jl} - \sum_{k \neq j,l} T_k) dt \right); \quad (29)$$

$$c_{jj(h)}(t) = c_{jj(h)}(0) \exp \left(-\frac{i}{\hbar} \int_0^t (E_{jj} - \sum_{k \neq j} T_k + T_j) dt \right). \quad (30)$$

If the system deforms slowly, the contributions of the type $c_{jl} \langle j | \partial/\partial t | l \rangle$ in Eqs. (25), (26) and (27) can be neglected and the solutions for seniority-two states given by the Lagrange method of variation of constants are:

$$c_{jl} = \frac{-i\hbar}{E_{jl} - \sum_{k \neq j,l} T_k - E_0 + \sum_k T_k} (v_j u_l - u_j v_l) \langle j | \frac{\partial}{\partial t} | l \rangle c_{0(h)} \quad (31)$$

$$c_{jj} = \frac{-i\hbar}{E_{jj} - \sum_{k \neq j} T_k + T_j - E_0 + \sum_k T_k} (u_j \dot{v}_j - v_j \dot{u}_j) c_{0(h)} \quad (32)$$

Finally the probabilities to find the system in a seniority-two state are:

$$|c_{jl}|^2 = \hbar^2 \frac{|v_j u_l - u_j v_l|^2 \langle j | \frac{\partial}{\partial t} | l \rangle^2}{(E_{jl} - \sum_{k \neq j,l} T_k - E_0 + \sum_k T_k)^2} |c_{0(h)}|^2 \quad (33)$$

$$|c_{jj}|^2 = \hbar^2 \frac{|u_j \dot{v}_j - v_j \dot{u}_j|^2}{(E_{jj} - \sum_{k \neq j} T_k + T_j - E_0 + \sum_k T_k)^2} |c_{0(h)}|^2 \quad (34)$$

The total energy of the system is;

$$\begin{aligned} E &= |c_0|^2 E_0 + \sum_{j,l} |c_{jl}|^2 E_{jl} \\ &= (1 - \sum_{j,l} |c_{jl}|^2) E_0 + \sum_{j,l} |c_{jl}|^2 E_{jl} \\ &= E_0 + \sum_{j,l} |c_{jl}|^2 (E_{jl} - E_0) \end{aligned} \quad (35)$$

If we consider that the collective kinetic energy is

$$\sum_{\nu,\mu} \frac{1}{2} B_{\nu\mu} \dot{q}_\nu \dot{q}_\mu = \sum_{j,l} |c_{jl}|^2 (E_{jl} - E_0), \quad (36)$$

and substituting the time derivative with derivatives with respect the generalized coordinates

$$\frac{\partial}{\partial t} = \sum_\nu \dot{q}_\nu \frac{\partial}{\partial q_\nu} \quad (37)$$

and considering that $|c_0|^2 \approx 1$ it follows

$$\begin{aligned} B_{\nu\mu} &= 2\hbar^2 \sum_{m,n \neq m} \frac{(E_{mn} - E_0) |v_m u_n - u_m v_n|^2 \langle m | \frac{\partial}{\partial q_\nu} | n \rangle \langle n | \frac{\partial}{\partial q_\mu} | m \rangle}{(E_{mn} - \sum_{k \neq m,n} T_k - E_0 + \sum_k T_k)^2} \\ &\quad + 2\hbar^2 \sum_m \frac{(E_{mm} - E_0) (u_m \frac{\partial v_m}{\partial q_\nu} - v_m \frac{\partial u_m}{\partial q_\nu}) (u_m \frac{\partial v_m^*}{\partial q_\mu} - v_m^* \frac{\partial u_m}{\partial q_\mu})}{(E_{mm} - \sum_{k \neq m} T_k + T_m - E_0 + \sum_k T_k)^2} \end{aligned} \quad (38)$$

where $B_{\nu\mu}$ are the effective mass parameter.

Using the identities

$$\langle i | \frac{\partial}{\partial t} | j \rangle = \frac{\langle i | \frac{\partial H}{\partial t} | j \rangle}{\epsilon_j - \epsilon_i} \quad (39)$$

the dependences with respect the derivative of the Hamiltonian are evidenced:

$$\begin{aligned} B_{\nu\mu} &= 2\hbar^2 \sum_{m,n \neq m} \frac{(E_{mn} - E_0) |v_m u_n - u_m v_n|^2 \langle m | \frac{\partial H}{\partial q_\nu} | n \rangle \langle n | \frac{\partial H}{\partial q_\mu} | m \rangle}{(E_{mn} - \sum_{k \neq m,n} T_k - E_0 + \sum_k T_k)^2 (\epsilon_m - \epsilon_n)^2} \\ &\quad + 2\hbar^2 \sum_m \frac{(E_{mm} - E_0) (u_m \frac{\partial v_m}{\partial q_\nu} - v_m \frac{\partial u_m}{\partial q_\nu}) (u_m \frac{\partial v_m^*}{\partial q_\mu} - v_m^* \frac{\partial u_m}{\partial q_\mu})}{(E_{mm} - \sum_{k \neq m} T_k + T_m - E_0 + \sum_k T_k)^2} \end{aligned} \quad (40)$$

The values of u_k and v_k are solutions of the time dependent pairing equations. Using notations (7), the expression (10) is eventually obtained.

It must be noticed that by replacing the energies with the approximate values [30] $E_{jl} \approx \sqrt{(\epsilon_j - \lambda)^2 + \Delta_{jl}^2} + \sqrt{(\epsilon_l - \lambda)^2 + \Delta_{jl}^2} = E_j + E_l$, by neglecting the differences between the sums of T_k -terms and by using the stationary values \tilde{v} and \tilde{u} in the previous expression, it is straightforward to obtain the usual cranking mass parameter for an adiabatic BCS state [5]. In this context, the following identities are also needed:

$$\tilde{u}_k^2 = \frac{1}{2} \left(1 + \frac{\epsilon_k - \lambda}{E_k} \right);$$

$$\begin{aligned}\tilde{v}_k^2 &= \frac{1}{2}\left(1 - \frac{\epsilon_k - \lambda}{E_k}\right); \\ \langle i | \frac{\partial H}{\partial q} | i \rangle &= \frac{\partial \epsilon_i}{\partial q}.\end{aligned}\tag{41}$$

- [1] Nix J R 1972 *Ann. Rev. Nucl. Sci.* **22** 65.
- [2] Inglis D R 1954 *Phys. Rev.* **96** 1059.
- [3] Inglis D R 1955 *Phys. Rev.* **97** 701.
- [4] Schutte G and Wilets L 1975 *Nucl. Phys. A* **252** 21.
- [5] Brack M, Damgaard J, Jensen A, Pauli H, Strutinsky V and Wong W 1972 *Rev. Mod. Phys.* **44** 320.
- [6] Iwamoto A and Greiner W 1979 *Z. Phys. A* **292** 301.
- [7] Iwamoto A and Maruhn J A 1979 *Z. Phys. A* **293** 315.
- [8] Schneider V, Maruhn J A and Greiner W 1986 *Z. Phys. A* **323** 111.
- [9] Liran S, Scheefer H J, Scheid W and Greiner W 1975 *Nucl. Phys. A* **248** 191.
- [10] Chaffin E F and Dickmann F 1976 *Phys. Rev. Lett* **37** 1738.
- [11] Koonin S E and Nix J R 1976 *Phys. Rev. C* **13** 209.
- [12] Blocki J and Flocard H 1976 *Nucl. Phys. A* **273** 45.
- [13] Mirea M, Tassan-Got L, Stephan C and Bacri C O 2004 *Nucl Phys. A* **735** 21.
- [14] Mirea M, Tassan-Got L, Stephan C, Bacri C O and Bobulescu R C 2007 *Phys. Rev. C* **76** 064608.
- [15] Mirea M arXiv:0907.1347v1; *Phys. Lett. B* submitted.
- [16] Mirea M 2008 *Phys. Rev. C* **78** 044618.
- [17] Davies K T R and Nix J R 1976 *Phys. Rev. C* **14** 1977.
- [18] Mirea M, Bajeat O, Clapier F, Ibrahim F, Mueller A C, Pauwels N and Proust J 2001 *Eur. Phys. J. A* **11** 59.
- [19] Mirea M 2006 *Nucl. Phys. A* **780** 13.
- [20] Maruhn J and Greiner W 1972 *Z. Phys.* **251** 431.
- [21] Mirea M, Tassan-Got L, Stephan C, Bacri C O, Stoica P and Bobulescu R C 2005 *J. Phys. G* **31** 1165.
- [22] Ledergerber T and Pauli H-C 1973 *Nucl Phys A* **207** 1.
- [23] Mirea M, Bobulescu R C and Marian P *Rom. Rep. Phys.* in print.
- [24] Diaz-Torres A, Gasques L R and Wiescher M 2007 *Phys. Lett B* **652** 255.
- [25] Hofmann H 1976 *Phys. Lett. B* **61** 423.
- [26] Brink D M and Weiguny A 1968 *Nucl. Phys. A* **120** 59.
- [27] Gozdz A, Pomorski K, Brack M and Werner E 1985 *Nucl Phys. A* **442** 26.
- [28] Baran A, Sheikh J A and Nazarewicz W 2009 *Int. J. Mod. Phys. E* **18**, 1054.
- [29] Dudek J, Dudek W, Ruchowska E and Skalski J 1980 *Z. Phys. A* **294** 341.
- [30] Ogle W, Wahlborn S, Piepenbring R and Fredriksson S 1971 *Rev. Mod. Phys* **42** 424.